Sec. 10.1 Composition of Functions

Composition of Functions Defined by Equations:
RECALL:

- The function $f(g(t))$ is said to be a **composition** of f with g.
- The function $f(g(t))$ is defined by using the output of the function g as the input to f.
- The function $f(g(t))$ is only defined for values in the domain of g whose $g(t)$ values are in the domain of f.

Ex: If f(x) = 2x + 6 and g(x) = 3x – 4, find f(g(x)).

$f(g(x)) = 2(3x-4)+6$
$= 6x-8+6$
$f(g(x)) = 6x-2$

Ex: Let $p(x) = \sin x + 1$ and $q(x) = x^2 - 3$. Find a formula in terms of x for $w(x) = p(p(q(x)))$.

$p(q(x)) = \sin(x^2-3)+1$
$p(p(q(x))) = \sin[\sin(x^2-3)+1]+1$

Compositions of Functions defined by Tables:

Ex. Complete the table. Assume that f(x) is invertible.

x	f(x)	g(x)	g(f(x))
0	2	3	2
1	3	1	3
2	1	2	1

$g(f(x)) = g(1)$
$g(1) = 1$
$\Rightarrow g(f(x)) = 1$

$g(2) = ?$
$g(f(x)) = g(2) = 2$
$\Rightarrow g(2) = 2$

Composition of Functions Defined by Graphs:

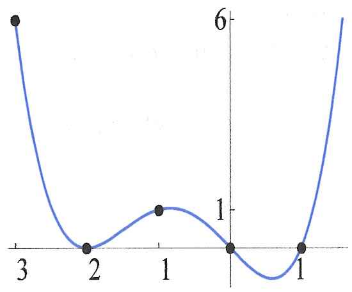
Ex. Let u and v be two functions defined by the graphs. Evaluate:

(a) $v(u(-1))$ (b) $u(v(5))$ (c) $v(u(0)) + u(v(4))$

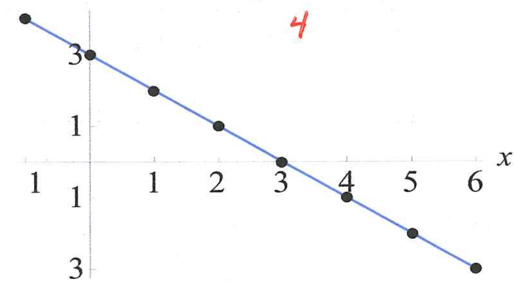
$v(1) = 2$ $u(-2) = 0$ $v(0) + u(-1)$

$3 + 1$

4



$u(x)$



$v(x)$

Decomposition of Functions:
When we reason backward to find the functions that went into the composition, it is called decomposition.

Ex: Let $h(x) = f(g(x)) = e^{x^2+1}$. Find possible formulas for $f(x)$ and $g(x)$.

$g(x) = x^2+1$ $f(x) = e^x$ or $f(x) = e^{x+1}$ $g(x) = x^2$

NOT: $f(x) = e^{x^2+1}$, $g(x) = x$ or $f(x) = x$, $g(x) = e^{x^2+1}$

TRIVIAL: $f = h$ or $g = h$

Ex: Let $p(z) = \sin^2(\ln z)$. Decompose $p(z)$ into three simpler functions by giving formulas for $f(z)$, $g(z)$ and $h(z)$ where $p(z) = f(g(h(z)))$.

$h(z) = \ln z$

$g(z) = \sin z$

$f(z) = z^2$

HW: pg 401-404 #3-6, 9, 12, 17, 19, 22, 24-26, 28, 34*, 39, 46, 48-50